Pedagogy for the 21st century; the Khan’s Academy solution for math class

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Abstract

A constant refrain from the American business community is that in order to compete in a worldwide economy within the 21st century the workforce needs to be better educated. The area needing the most significant improvement has been touted as the gap found in the math and science scores of our youth. Educators have expressed that the pedagogy of their profession has not kept pace with the times and they have borne the brunt of the blame without the benefit of new resources for this task. In order to connect to the student we can utilize an “adaptive assessment environment” to close this divide. Utilizing the vast improvements in rapid adaptions to various students’ capabilities, Khan’s Academy has developed technologies that can guide and assist a coach or teacher to better educate the student of the 21st century.
Pedagogy for the 21st century; the Khan’s Academy solution for math class

Educators acknowledge that the pedagogy has not had a reboot since the time of Aristotle. Consequently technology has stepped into the fray and appears to have taken hundreds of thousands of willing students to sign up and engage in a fruitful exchange that could possibly alter the future of education. The premise of the effectiveness of these Massive open online courses (MOOCs) need to be evaluated.

The Research Problem

Students frequently complain that they are bored, uninterested and otherwise not engaged in the courses they fail. The latest pedagogy shift argues that a digital (computer) program or course cannot offer the “connectivism…interaction and dialogue” that a learner needs to succeed. (Clara M., 2013) Observing the extreme growth in the availability of MOOC’s seems to demonstrate that these courses are successfully benefitting the student that completes the courses or lessons made available to him or her, on the one hand. On the other hand, it is common knowledge that the proficiencies of high school students in the United States in math and science have significantly dropped in rank as compared to their counterparts around the world. In this highly competitive environment Khan’s Academy has stepped in as a solution to deliver significantly better educated students via the latest “adaptive assessment environment” for the 21st century.

As American students of the 21st century are all too willing to read about the latest trending topics on their smartphone or other digital device; they oftentimes fail to engage in their coursework. Previous analysis of online learning or distance education has shown that these arenas did not significantly educate the student unless the learner was highly motivated to begin with. Yet, like most all areas that can be considered a growth industry, the world of academia has
been ripe with the latest trends in course builders, adaptive assessment environments and numerous other online learning innovations. Big business has joined to partner with elite universities to create Coursera and Udacity, Inc. (Arnold, 2013) While Bill Gates gushed about Khan’s Academy as a source he sent his own children to, he later helped fund the expansion and growth of this MOOC- like classes.

Looking overseas to our highly successful competition one thing was apparent; they have adapted to the 21st century model of open source, free online instruction. One of the first alternative online learning services was Mathigon. Developed in England, “Mathigon is interactive, fresh and indicative of the innovation that occurs when individuals with knowledge, motivation and technical resources give mathematics instruction a ground-up rebuild.” (Arnold, 2013) Khan’s Academy follows this model, but has branched out to offer much more than math classes.

Dr. Pritchard states in her editorial, MOOC’s: An Opportunity for Innovation and Research, “It’s rare to have such a clear-cut transition to a new service that spreads so rapidly across so many institutions, and that offers such challenges to our previous ways of doing business.” (Pritchard, 2013, p. 127) That is the business of education. There is a lot of money to be made in education as well as the loss of potential future monies; so criticism abounds. As many question the benefits provided by MOOC’s some fear that these sources have not been properly vetted due to the fact that there is the potential for droves of students to register for free courses rather than borrow thousands to attend a traditional college or university setting, especially if they are given college credit. A new bill was proposed in California to do exactly that. “The proposed legislation, SB 520, would require state colleges and universities to grant credit to students who, unable to register for core classes at their home universities…, register for
massive open online courses (MOOCs) instead.” (Busch, 2013, p. 9) In his analysis, Vardi points to the significance of “450,000 students signing up for (free) computer science classes offered by Stanford University in the fall of 2011” as the turning point in this debate. (Vardi, 2012, p. 5)

The purpose of this research study is to examine the progress made by fifth grade students following the inclusion of access to Khan’s Academy adaptive assessment environment in their math classes. This program utilizes the latest adaptive intelligent technology for web based educational courseware to provide the student with direct individualized instruction, tutorials and classes. It provides the student, coach and teacher with statistics to analyze where the student may need help or direction.

The research question to be examined is to what extent Khan’s Academy materials and courseware in the classroom improves student scores in math assessments? By examining the promotional information as well empirical evidence provided, it seems like this method of tutoring and instructing students is effective as it incorporates computer programming of many diverse assessment tools that historically took the educator or instructor days to evaluate. The capacity of computer programs to analyze data is vital to this adaptation.

Hypothetically it can be predicted that the students will demonstrate a significant improvement in math proficiencies in every area addressed through Khan’s Academy. Fifth grade students will be evaluated as this has been a turning point in many math students future. Many middle school campuses or junior high schools have adopted either a pre-algebra curriculum or maintained a general math class. This appears to be the time when a math student has been effectively taught the course or has been lost to a future of struggling with the concepts.
Justification

While the United States is considered home to the best in education and the most successful universities, the ability to claim that for a fact in the future is at stake. The latest venture into a MOOC platform is FutureLearn, a collaboration of over twenty top universities founded in the United Kingdom. Numerous students are registering for classes on these platforms and we need to be a part of this trend. In order to do this in the most effective manner, we need to analyze and assess the benefits and the technologies inherent in the distribution of the program(s).

If improvements in math scores can be achieved the results of this study could be utilized by education planners and facilitators. The possibilities for a better educated student entering higher education and the workforce would increase dramatically and along with that the funding sources, tax base and all the benefits that these elements can provide.

Definition of terms

For the purposes of this study, the lists of terms below are defined as follows:

**MOOC**: massive open online courseware

**Connectivism**: theory of learning which emphasizes the role of the social and cultural context definition.

**Tutorial**: a method of transferring knowledge and may be used as a part of a learning process. More interactive and specific than a book or a lecture; a tutorial seeks to teach by example and supply the information to complete a certain task.
Review of the Literature

Despite the concerns of academia, true research into the capability of Khan’s Academy, MOOC’s (massive open online courses) and similar projects is sparse. At the Khan Academy’s actual website, https://www.khanacademy.org/, the organization lists numerous Case Studies, however these seem to be in house video testimonials of the effectiveness of the implementation of what Khan’s Academy refers to as their online learning resource. In a report featured in the coaches’ resources the academy refers to numerous educational sources documenting the effectiveness of the practices implemented in their program. The section entitled Research foundations details the fact that their philosophy is based on mastery based learning, personalized instruction and interactive and exploratory learning. (Research foundations, 2012) In certain aspects the program appears to promote what is commonly referred to as an adaptive learning environment. However, in this research report the Academy breaks down the approach in a different manner, highlighting a multifaceted philosophy that utilizes data that seems to promote their principles. While online learning in general has been researched in great detail, the authors refer to a US Department of Education meta-analysis of numerous studies touting the benefits and advantages students made when provided with online instruction. Next, the report specifies the mastery based learning approach to instruction which is incorporated into the Khan Academy solution. This is the concept that prescribes that the student must accomplish certain tasks, followed by assessments and evidence of the student’s comprehension of the material prior to delving into the next concept or section. There are numerous references detailing the effectiveness of this type of teaching strategy included in the research article. The Personalized Learning section also referred to as Personal System of Instruction (PSI) highlights a meta-analysis of over 75 comparative studies revealing “superior student achievement” via this
method. (Research foundations, 2012, p. 2) In the report the authors refer to targeted tutoring methods and direct immediate support from the coach or instructor that may not be available in traditional settings. In emphasizing the feedback provided by an online learning environment with a Personal System of Instruction, one can broaden the criteria for data to include a number of other research studies and sources that complement this foundational criterion.

The final component of the Khan Academy learning approach is interactive and exploratory learning, which can reference the peer collaborative projects as well as the tutoring; where those that have mastered certain modules assist those that are struggling with key concepts.

Quite a number of elements in what is considered collaborative learning has been analyzed as an effective method of instruction. In the journal article by Zheng, Yang, Cheng and Huang; the group analyzed online learning environments around the world to determine not only the effectiveness of these communities but to determine if there were any crucial differences and if so what information could be drawn from their analysis. As detailed in the Khan Academy report the strategies utilized by these methods is infusing a sense of community into the learning process by allowing group activities and a sense of immediate feedback in the form of peer review similar to social networking sites. The group focuses on the fact that younger students expect quick responses to work that they do and that the element of expectation is apparent in most effectual programs used to instruct these students. Assessment is instrumental to a positive benefit for collaborative learning techniques and the analysis defines the methods by which the program embeds the assessment into the group learning activities after motivating the student to achieve a learning objective. Evidently this process is becoming universal as these methods seem to be common in the East as well as the West.
The problems with older methods of assessments are underscored in a conference proceeding that examined the nature of most current forms of assessments in order to guide students’ progress. In this report, Mislevy, Almond and Lukas (2004) examine multiple models and the design framework of standardized tests, classroom quizzes and the latest simulation-based assessments as well as student portfolios and student-tutor interaction. Emphasizing the fact that there has been an accumulation of data for quite a number of years of the older standardized testing models, newer assessments can parse data at multiple levels and provide more specific criteria in order to guide the student and evaluate his/her progress. The previous off-the-shelf assessments needed to provide broader evaluations due to the fact that information was across the curve from low scores to superior results. On a wide scale analysis these results do little to determine what was or has been missing from the instruction provided to the students. (Mislevy, et al, 2004) The latest adaptive assessment technology has been touted as eliminating the guesswork formally used in the analysis of these types of wide ranging results.

Salman Khan, (2013) the developer of the Khan Academy, himself details the benefits of alternative assessments. In his article Khan explains that student’s expectations are very different than what they were years ago. Businesses as well, expect a more engaged employee with skills that are developed and proven as opposed to credentials that are solely based on a college transcript. He actually promotes an internship method of higher education that he states would provide for rigorous assessments of skills in a self-paced academic environment based on the EdX MOOC or even his Khan Academy which currently primarily caters to the younger student (Khan S. , What College Could Be Like, 2013).

In an interesting journal article, Kronholz (2012) identifies the components of a personalized learning program that have been proven most effective in various classroom
settings ranging from a high performance elite classroom to a grade level or low performing school campus in which the Khan Academy math modules were incorporated in a blended classroom fashion. The assessments utilized typical benchmarking and then the California Standards Test over the course of the program’s assessment period and revealed substantial improvements. The author examined and explained apparent shortcomings as well as obvious successes. The comparative method the school districts implemented in their trial period is integral to determining that the data generated by the program is unbiased and proving to be successful in improving the students’ performance in the classroom.

Another analysis of the Khan Academy math modules section was described by Khan and Slavitt (2013). The emphasis underscored here is on the particular benefits of self-paced learning, collaborative learning, and the innovations provided in newer educational technologies. The authors point to a pilot program, again in California, where the students were empowered to forge ahead or got help from their teacher when they were stuck on concepts they could not grasp. The access to their data is what the authors believed enabled the students to succeed. (Khan & Slavitt, 2013)

While little to nothing could be found in numerous searches of specific research related to Khan’s Academy explicitly as detailed above, there are many elements that the academy professes to be part and parcel of their educational philosophy that have been analyzed and researched in greater volumes. Another key feature would be the open online learning aspect of the program. The single aspect of Bonk’s (2009) research and analysis is that the layer of open access to the code which allows numerous people, professionals and educators to contribute to the final elements which now results in a perfected product. He proclaims that he believes the information becomes masterful as so many across the globe contribute to these portals in which
WE-ALL-LEARN. Several researchers seem to agree with this premise as noted in the highly technical meta-evaluations by Williams (2010) whose research includes data that was collected and analyzed then judged and evaluated for specific criteria on several levels. The study looked at cost effectiveness, completion rates, student satisfaction, student learning, and student engagement as well as numerous measures important to administrators. The scope of the research included multiple levels of context such as activity, course, program and institution.

As relevant to the program being examined, what is not as well understood is the latest available technology that contributes to the adaptive assessments made in these newer incarnations of online learning programs. When the academy first began, it simply utilized an online repository of prerecorded math concepts videos uploaded to a standard public YouTube channel. Following Salman Khan’s growing popularity and acknowledgment by Bill Gates and other influential technology giants, Khan received many large grants to expand his organization. He developed his website and hired others to assist him in developing resources, curriculum guides and exercises that are incorporated into the ever expanding subject matter. They conducted research through numerous pilot programs in school districts across the nation. One pilot by Ash (2013) concluded that the experiment was highly successful and the school districts throughout the state were ready to follow suite as well as expand their efforts to include additional subject areas. The personalized learning was determined to be identified as a best practice that allowed for the participants to bolster their performance in skills that were all previously struggling to master.

In lieu of these numerous pilot programs and feedback received via comments, notes and emails the academy has incorporated the Common Core Curriculum Resources which are being utilized in about forty (40) of the fifty contiguous United States. The web site has added math
practice problems that are aligned with specific standards assigned to appropriate grade levels. The student content is peer-reviewed by specialists in the field, such as math, science, art and history. The program is worldwide and translated into numerous other languages as well (Webley, 2012). The common perceived benefit of this model seems to be that the student’s personalized learning allows for them to fill in gaps in foundational understanding which leads to mastery of the concepts or subject. The researchers believe that as the student masters the subject, his/her confidence is built and he/she becomes more engaged in their learning. These points of agreement can be seen in additional research such as those identified in the reports generated by analysis of student driven curriculum designs and variations on the framework. Other educators in the field have designed courses that allow for computer simulated science labs, and generated experiments through completely computer generated simulations. The students were allowed to perform, evaluate and modify their work which in final analysis improved student’s comprehension of the subject matter in every area assessed (Khan, 2011).

In conclusion one can determine that while specific information concerning a viable research study of the benefits of implementation of Khan Academy’s resources is imminently needed, the overall benefits of many of the specific elements contained in this evolving model are available. Khan Academy seems to have made changes in their model following the influx of cash and grants, feedback from pilot studies at the various campuses and letters, email and comments from the end users and their coaches. Parents, teachers and professionals have come together to help make changes that do incorporate many of the best practices identified in the latest pedagogy which will hopefully benefit the students that access the website.
Methodology for: Pedagogy for the 21st century; the Khan’s Academy solution for math class

As stated the purpose of this research study would be to examine the progress made by fifth grade students following the inclusion of access to Khan’s Academy adaptive assessment environment into their math classes. The researcher would utilize the latest adaptive intelligent technology for web based educational courseware to provide student participants with direct individualized instruction, tutorials and classes. As part of the treatment students, coaches and teachers will be provided with statistics to analyze where students may need help or direction.

Research Methodology includes a review of the research method and design appropriateness, and a discussion of the population and sample.

Research Method and Design Appropriateness

Throughout the course of the research project results will be examined as they are collected over the course of an academic school year. This design would be a comparative case study of two groups chosen from the fifth grade class level. One group, classified as the experimental group would have a new computer program added to their math curriculum; the Khan Academy World of Math program. The other group would serve as a control, being taught the traditional, standard math curriculum.

Participants will be selected from six fifth grade classrooms that would agree to participate in this study. The ideal number of participants would be approximately 150 students in which 75 students from three classrooms would agree to include tutorial classes via the Khan Academy web site and 75 students, or three class groups with one control group, taught the standard curriculum.
These six fifth grade class groups would be given pretests via the standard Math Benchmark prior to the start of the tutorial phase.

The three fifth grade class groups that add the Khan Academy program will be provided access to a computer lab with internet access.

At the end of the school year, all the students again will be assessed with a posttest utilizing the standard Math Achievement test.

Gains made by both groups will be measured and any deviations and/or differences will be plotted.

The students in the control group will be taught utilizing traditional methods covering the standard requirements for a fifth grade math student in the school year. The students in the experimental group will be following standard Khan Academy methods of instruction.

**Research Questions**

The research question to be examined is to what extent Khan’s Academy materials and courseware in the classroom improves student scores in math assessments? By examining the promotional information as well empirical evidence provided, it seems like this method of tutoring and instructing students is effective, as it incorporates computer programming of many diverse assessment tools that historically took the educator or instructor days to evaluate. The capacity of computer programs to analyze data is vital to this adaptation.

**Population**

The population will be fifth grade students whose range in age from approximately nine to twelve years of age, with an average of ten to eleven years of age.
The population of the school district includes:

Student Profile

<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethnicity (Hispanic)</td>
<td>48,519</td>
<td>99%</td>
</tr>
<tr>
<td>Economically Disadvantaged</td>
<td>47,181</td>
<td>96%</td>
</tr>
<tr>
<td>At Risk</td>
<td>28,961</td>
<td>59%</td>
</tr>
<tr>
<td>Limited English Proficient</td>
<td>14,750</td>
<td>30%</td>
</tr>
<tr>
<td>Migrant</td>
<td>1,256</td>
<td>3%</td>
</tr>
</tbody>
</table>

Population Sample

The local district has an enrollment of approximately 3,450 fifth grade students overall in thirty seven (37) elementary school campuses. The average class size is approximately (25) twenty five students per class with a range of two to four fifth grade classes per campus.

A convenience sampling of six fifth grades will be selected to participate, including the three control classrooms which will be pretested with the Standard Math Benchmark for the fifth grade level and post tested via a Standard Math Achievement test at the end of the school year.

Informed Consent and Confidentiality

Participants will be provided with a voluntary informed consent form.

Participants will be informed that they are being asked to participate in a research study,

Participants will be provided an explanation of the purposes of the research and the expected duration of their participation,

Participants will be given a description of the procedures to be followed and of any procedures that must be identified,

Participants will be given a description of any benefits to themselves or others that may reasonably be expected from the results of the study,
Participants will be given a statement describing the extent, to which confidentiality of records identifying the subject/participant will be maintained.

Participants will be told to contact, the researcher for answers to pertinent questions about the research and research subjects'/participants' rights, and whom to contact in the event of a research-related outcome of concern.

Participants will be given a statement that participation is voluntary, refusal to participate will involve no penalty or loss of benefits to which the subject/participants is otherwise entitled, and the subject/participant may discontinue participation at any time without penalty or loss of benefits to which the subject/participant is otherwise entitled.

**Data Collection**

Data will be collected in the form of achievement score from pretests, posttests and interim reports generated via the Khan Academy website.

Any logs, reports and any other narratives from the participating classrooms provided during the duration or course of the study may be included in the collection of data.

**Instrumentation**

Standardized Math Benchmark tests for the fifth grade student will be utilized as the pretest form prior to any tutorial phase. Please see Appendix A for an example.

A Standard Math Achievement test will serve as the posttest to measure gains (or losses) at the end of the academic year and tutoring phase.

There are several reports provided via the Khan website such as the following report:

<table>
<thead>
<tr>
<th>Student</th>
<th>Struggling Needs</th>
<th>Practiced</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Mastered</th>
<th>Time Sper</th>
<th>Time Sper</th>
<th>Time Sper</th>
<th>Time Sper</th>
<th>Badges Earned</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>JamesLara</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>JoeLara</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>JakeLara</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
This data may be collected and utilized as a guide or an assessment if deemed appropriate.

Please see the Appendix D through Appendix F for details.

Validity and Reliability

The standardized Math Benchmark test for fifth grade students has long been used in the school district. The Math Achievement test is used nationwide for purposes of determining the effect of new or different teaching methods. Any anomalies may be attributed to the Khan Academy program, so I will be track these via the charts and reports generated by the Khan program.

Data Analysis

A comparison of any gains, losses, differences and deviations will be charted and analyzed following the posttests. Math posttest scores of students in control groups will be compared against Math posttest scores of students in the treatment group by running a T-Student group comparison analysis.

Summary

The students found within the school district are primarily economically disadvantaged, minority students that typically do very poorly in math and science classes. Any attempt to improve the instruction method, increase retention and improve scores would be welcomed by the school district, parents and students.

The inclusion of the Khan Academy website’s *World of Math* has been utilized in many campuses throughout the nation and the world as well. There have been several narrative case
studies done that do not include pertinent data such as actual scores, gains and other vital information.
Mathematics

Part 1

1. Sam's teacher has a rectangular prism in a bag. She is giving Sam clues to help him determine if the rectangular prism is also a cube. The clues Sam's teacher has given him are listed below.

- The width of the prism is 2 inches.
- The height of the prism is 2 inches.
- The prism has a volume of 8 cubic inches.
- The prism has 6 faces and 8 vertices.

Which clue is not needed to determine if the rectangular prism is a cube?

A. The width of the prism is 2 inches.
B. The height of the prism is 2 inches.
C. The prism has a volume of 8 cubic inches.
D. The prism has 6 faces and 8 vertices.
2 Monica used three different colors to paint a rectangle.

- $\frac{1}{2}$ of the rectangle is painted blue.
- $\frac{1}{6}$ of the rectangle is painted yellow.
- The rest of the rectangle is painted green.

What fraction of the rectangle is painted green?

F $\frac{3}{4}$
G $\frac{2}{3}$
H $\frac{1}{3}$
J $\frac{1}{12}$

3 A package of 50 balloons was shared by 6 people.

- Each person received 8 balloons.
- There were 2 balloons left over.

Based on this information, which statement is not true?

A Out of the package of 50 balloons, people received 48 of the balloons.
B Denise has $\frac{2}{8}$ of the balloons she needs in order for each person to have 9 balloons.
C In order for each person to receive another balloon, Denise needs 4 more balloons.
D There are 2 leftover balloons because the package of 50 balloons cannot be divided by 6 evenly.
4. What is the value of the expression below?

\[ 18 + 9 \div 3 - 2 \times 4 - \frac{3}{8} \]

F. \( \frac{27}{8} \)
G. \( \frac{13}{8} \)
H. \( \frac{12}{8} \)
J. \( \frac{7}{8} \)

5. Which statement is true?

A. \( 140.02 = 104.02 \)
B. \( 140.02 < 104.2 \)
C. \( 104.02 = 104.2 \)
D. \( 104.02 < 140.2 \)

Go On ➤
The triangular sign shown below has a height of 14 inches and a base length of 18 inches.

Area = \( \frac{1}{2} \) base \( \times \) height

What is the area of this sign?

- **F** 126 square inches
- **G** 252 square inches
- **H** 504 square inches
- **J** 756 square inches
7 Carmen used a cart to deliver packages.

- The packages she delivered weighed a total of 574 pounds.
- She carried a total of 50 pounds or less in her cart on each trip she made.

What is the fewest number of trips Carmen could have made to deliver all of these packages?

- **A** 9
- **B** 10
- **C** 11
- **D** 12

8 What is the value of the expression below, when $\rho = \frac{1}{4}$?

$$\frac{9}{4} + \left(\frac{5}{8} - \frac{3}{8}\right) + \rho$$

- **F** 11
- **G** $10\frac{1}{4}$
- **H** 10
- **J** $9\frac{3}{4}$
Danny used a triangle, two congruent rectangles, and two congruent squares to create the design shown below. The dimensions of the design are shown in inches.

Area of rectangle = length × width

Area of triangle = \( \frac{1}{2} \times \text{base} \times \text{height} \)

What is the combined area of the shaded parts and the unshaded part?

- A 108 square inches
- B 131 square inches
- C 140 square inches
- D 172 square inches
A nickel has a thickness of 1.95 millimeters. Which number is equivalent to 1.95?

- F \( \frac{19}{20} \)
- G \( \frac{9}{5} \)
- H \( \frac{1}{5} \)
- J \( \frac{19}{200} \)
The table below shows the amount of time Braden spent reading each day for five days.

<table>
<thead>
<tr>
<th>Day</th>
<th>Amount of Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>1 3/4</td>
</tr>
<tr>
<td>Tuesday</td>
<td>2 1/6</td>
</tr>
<tr>
<td>Wednesday</td>
<td>1 9/10</td>
</tr>
<tr>
<td>Thursday</td>
<td>5 7/60</td>
</tr>
<tr>
<td>Friday</td>
<td>2 2/3</td>
</tr>
</tbody>
</table>

Which estimate is closest to the total number of hours Braden spent reading on these five days?

A  6 hours
B  7 hours
C  10 hours
D  14 hours
The table below shows the populations of five Tennessee cities in 2006.

<table>
<thead>
<tr>
<th>City</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jackson</td>
<td>62,711</td>
</tr>
<tr>
<td>Clarksville</td>
<td>113,175</td>
</tr>
<tr>
<td>Bristol</td>
<td>25,351</td>
</tr>
<tr>
<td>Memphis</td>
<td>670,902</td>
</tr>
<tr>
<td>Franklin</td>
<td>55,870</td>
</tr>
</tbody>
</table>

What is the median of these populations?

F 62,711
G 55,870
H 37,360
J 25,351
Freddie’s family packed three suitcases for a vacation. The weight of each suitcase is listed below.

$24\frac{13}{16}$ pounds, $34.25$ pounds, $41\frac{5}{8}$ pounds

What is the combined weight of these three suitcases?

A $100\frac{67}{80}$ pounds

B $100\frac{11}{16}$ pounds

C $100\frac{3}{16}$ pounds

D $99\frac{19}{28}$ pounds

What value of $y$ makes this statement true?

$y - 9\frac{1}{7} = 10$

F $\frac{6}{7}$

G $9\frac{6}{7}$

H $19$

J $19\frac{1}{7}$
A coordinate grid is shown below.

A line segment that is $4\frac{1}{2}$ units long will be drawn on the grid. Which list of coordinate pairs could represent the locations of the endpoints of this line segment?

A  $(1, 2), \left(5\frac{1}{2}, 2\right)$

B  $\left(1, 3\frac{1}{2}\right), \left(1, 7\frac{1}{2}\right)$

C  $\left(2\frac{1}{2}, 2\frac{1}{2}\right), \left(6\frac{1}{2}, 2\frac{1}{2}\right)$

D  $(2, 6), \left(2\frac{1}{2}, 6\right)$
Which number is equivalent to the number written below?

\[10 \frac{4}{25}\]

F  \[10 \frac{16}{10}\]

G  \[10 \frac{425}{1,000}\]

H  10.16

J  10.016
The graph below shows the numbers of customers at a restaurant at the ends of five different hours.

Which table is best represented by this graph?

<table>
<thead>
<tr>
<th>Hour</th>
<th>Number of Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hour</th>
<th>Number of Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hour</th>
<th>Number of Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
</tr>
<tr>
<td>2</td>
<td>51</td>
</tr>
<tr>
<td>3</td>
<td>98</td>
</tr>
<tr>
<td>4</td>
<td>99</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
</tr>
</tbody>
</table>
18 Jana has 125 songs on her music player.

- She puts these songs on playlists, with 20 songs on each playlist.
- She does not put the same song on more than one playlist.

After Jana makes the greatest number of playlists possible, which statement is true?

F Jana has \( \frac{5}{20} \) of the songs she needs to make another playlist.

G Jana has 15 of the songs she needs to make another playlist.

H Jana has \( \frac{5}{125} \) of the songs she needs to make another playlist.

J Jana has 6 of the songs she needs to make another playlist.

19 The width of a human hair is about twenty-five millionths of a meter. How is twenty-five millionths written in standard form?

A 2,500,000

B 25,000

C 0.00025

D 0.000025
20 The table below shows the number of students in each of six classrooms.

<table>
<thead>
<tr>
<th>Classroom</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
</tr>
<tr>
<td>F</td>
<td>19</td>
</tr>
</tbody>
</table>

What is the mean number of students in these classrooms?

F 7
G 18
H 19
J 108

21 Solve:

\[90 \frac{2}{3} + 16 \frac{5}{18} =\]

A 107 \frac{1}{6}
B 106 \frac{17}{18}
C 106 \frac{7}{18}
D 106 \frac{1}{3}
22 Look at the inequality below.

\[5 \cdot n < 30\]

Which set contains only values of \( n \) that make this inequality true?

- **F** \{25, 28, 29\}
- **G** \{22, 23, 24\}
- **H** \{4, 5, 6\}
- **J** \{1, 4, 5\}

23 A three-dimensional figure is shown below.

- Each triangular face has an area of 28.29 square centimeters.
- Each rectangular face has an area of 72 square centimeters.

What is the total surface area of this figure?

- **A** 272.58 square centimeters
- **B** 244.29 square centimeters
- **C** 228.87 square centimeters
- **D** 100.29 square centimeters
24 Ms. Flores climbed all 897 steps at the Washington Monument in 3 hours. She climbed the same number of steps each hour. What is the total number of steps Ms. Flores climbed each hour?

F 232
G 290
H 299
J 366

25 The list below shows the weights, in pounds, of nine babies in a nursery.

\[ \frac{8}{4}, 6, \frac{7}{4}, \frac{13}{16}, 9, \frac{8}{4}, 6, \frac{8}{4}, \frac{1}{8} \]

What is the mode of these weights?

A 3 pounds
B 6 pounds
C \( \frac{8}{8} \) pounds
D \( \frac{3}{4} \) pounds

Go On
26. Theo drew the figure below.

![Diagram of a rectangle and a triangle]

Area of rectangle = length \times width

Area of triangle = \frac{1}{2} \times base \times height

What is the area of the figure?

- **F** 12 sq cm
- **G** 48 sq cm
- **H** 60 sq cm
- **J** 72 sq cm

27. Which inequality is true?

- **A** 1,580 > 158.0
- **B** 1,580 > 15,800
- **C** 158.00 > 158.0
- **D** 158.00 > 1,580
28. What is the value of the expression below?

$$\frac{3}{5} + 36 \div 2 \cdot 3 - \frac{1}{10}$$

F 6 $\frac{1}{2}$

G 53 $\frac{9}{10}$

H 54 $\frac{2}{5}$

J 54 $\frac{1}{2}$
Part 2

29. The top face of a box shaped like a parallelogram is shown below.

```
8 inches
10 inches
```

```
6 inches
```

Area = base × height

What is the area of the top face of the box?

A 16 square inches  
B 30 square inches  
C 60 square inches  
D 80 square inches
30. There are 117 visitors waiting to be put into groups at a museum. Each group can have 14 visitors at most. What is the fewest number of groups that will be needed for these visitors?

F 8
G 9
H 12
J 13

31. What is the value of the expression below, when \( r = 8.05 \)?

\[
(90.5 - r) + (5 \times 20) + r
\]

A 1,757.05
B 222.7
C 190.5
D 57.8
The model below is shaded to represent a number.

Which number cannot be represented by this model?

F 2.5
G \frac{10}{4}
H \frac{24}{8}
J \frac{20}{5}
Ben woke up at 7:00 A.M.

- He spent exactly 8 hours at work.
- He started working on his computer at 8:45 A.M.
- He arrived home at 6:45 P.M.

What other information is needed to determine the amount of time Ben takes to get home from work?

A the time at which Ben left work to go home
B the time at which Ben left home to go to work
C the time at which Ben started getting ready for work
D the time at which Ben stopped working on his computer

Solve:

$$16.384 - 8\frac{3}{4} =$$

F 8.434
G 8.044
H 7.634
J 7.534

Go On ▶
35. What value of \( t \) makes this statement true?

\[ t - 3 \frac{5}{6} = 9 \frac{2}{6} \]

A. 13\( \frac{1}{6} \)
B. 12\( \frac{1}{6} \)
C. 6\( \frac{1}{2} \)
D. 5\( \frac{1}{2} \)

36. The Tesoro family drank 3.4 gallons of milk in one week. Which number is equivalent to 3.4?

F. \( \frac{34}{1} \)
G. 3.40
H. 3.04
J. \( \frac{34}{100} \)
Mrs. Martin cut a pineapple into 18 equal pieces.

- She gave each of her 4 children an equal number of the pieces.
- Mrs. Martin got the leftover pieces.
- Mrs. Martin got fewer pieces than each child.

Based on this information, which statement could be true?

A  Mrs. Martin got \( \frac{2}{18} \) of a piece.
B  Mrs. Martin got 16 pieces.
C  Mrs. Martin got 0.5 of a piece.
D  Mrs. Martin got 2 pieces.
The coordinate grid below shows the locations of five dogs at a dog park.

Which two dogs appear to be 3 units apart?

- **F** Max and Jed
- **G** Max and Shiloh
- **H** Hope and Jed
- **J** Hope and Buckley
A unit of mass, called a slug, can be converted to kilograms by multiplying by the number below.

Fourteen and five thousand, nine hundred thirty-nine ten thousandths

How is this number written in standard form?

A 14,005,939
B 145,939
C 14.5939
D 14.05939
The table below shows the numbers of hours Gerald spent using different types of electronic devices during one week.

<table>
<thead>
<tr>
<th>Type of Device</th>
<th>Number of Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video Game</td>
<td>5 1/2</td>
</tr>
<tr>
<td>Computer</td>
<td>5 3/4</td>
</tr>
<tr>
<td>Music Player</td>
<td>4 1/4</td>
</tr>
<tr>
<td>DVD Player</td>
<td>3 1/2</td>
</tr>
</tbody>
</table>

Which graph best represents the data in the table?
Rebecca drew a design with two triangles and three congruent rectangles, as shown below.

What is the perimeter of Rebecca’s design?

A  150 cm  
B  102 cm  
C  82 cm  
D  50 cm
Two teams, the Hawks and Eagles, played in a basketball game.

- \(\frac{5}{8}\) of the players on the Hawks team scored points in the game.
- \(\frac{3}{4}\) of the players on the Eagles team scored points in the game.

What is the difference between the fractions of players who scored points on these two teams?

- **F** \(\frac{11}{8}\)
- **G** \(\frac{2}{3}\)
- **H** \(\frac{1}{2}\)
- **J** \(\frac{1}{8}\)

David has two ropes, with the lengths listed below.

- The first rope has a length of \(19\frac{1}{8}\) inches.
- The second rope has a length of \(39\frac{3}{4}\) inches.

Which estimate is closest to the difference between the lengths of David's ropes?

- **A** 10 inches
- **B** 21 inches
- **C** 30 inches
- **D** 61 inches
44 What is the value of the expression below?

\[ 10 \times \left( \frac{5}{10} - \frac{1}{2} \right) + 6 \]

F 6
G 7
H 10 1/2
J 11 1/2

45 The inequality below shows how Keaton’s age compares with two other ages.

\[ \frac{12}{2} \text{ years} > \text{Keaton’s age} > \frac{11}{4} \text{ years} \]

Which age could be Keaton’s age?
A 12.75 years
B 12.5 years
C 11.5 years
D 11.25 years
46. What value of \( n \) makes this statement true?

\[
48 \frac{1}{6} + n = 102 \frac{2}{3}
\]

\[
\begin{align*}
F & \quad 54 \frac{1}{3} \\
G & \quad 54 \frac{1}{2} \\
H & \quad 146 \frac{1}{3} \\
J & \quad 150 \frac{5}{6}
\end{align*}
\]

47. There are 106 children going on a field trip. The children will be put into groups of 6 or fewer students. Each group of students must have 1 adult supervisor. What is the fewest number of adult supervisors needed for this field trip?

\[
\begin{align*}
A & \quad 18 \\
B & \quad 17 \\
C & \quad 14 \\
D & \quad 11
\end{align*}
\]
48 The square pyramid below has a height of 11 centimeters.

![Pyramid Diagram]

\[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]

What is the volume of the square pyramid?

- **F** 22 cm³
- **G** 44 cm³
- **H** 132 cm³
- **J** 396 cm³
A set of numbers is shown below.

\[ \{78, 80, 82\} \]

Which inequality is true, when \( x \) is replaced by each of the numbers in this set?

A  \( x < 82 \)
B  \( x < 90 \)
C  \( x > 82 \)
D  \( x > 90 \)

A worker spent $564 on supplies to remodel 12 rooms in an office. The worker spent the same amount of money to remodel each room. What is the total amount of money the worker spent to remodel each room?

F  $40
G  $41
H  $42
J  $47

What is the value of the expression below, when \( k = 3.7 \)?

\[ 25(k + 10.3) - k \]

A  350
B  346.3
C  257.5
D  99.1
52 Five points are labeled on the coordinate grid below.

Which two points appear to be 1.25 units apart?

- F Point R and Point S
- G Point S and Point U
- H Point T and Point V
- J Point U and Point V
53. Which number is equivalent to 23.017?

A. \( 23 \frac{17}{1} \)

B. \( 23 \frac{17}{10} \)

C. \( 23 \frac{17}{100} \)

D. \( 23 \frac{17}{1,000} \)
The graph below shows the number of each type of ticket sold at a water park on Saturday.

<table>
<thead>
<tr>
<th>Water Park Tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Child – 1 day</td>
</tr>
<tr>
<td>Adult – 1 day</td>
</tr>
<tr>
<td>Child – Season Pass</td>
</tr>
<tr>
<td>Adult – Season Pass</td>
</tr>
</tbody>
</table>

Key: ■ = 30 tickets

Which table of information is best represented by this graph?

**F**

<table>
<thead>
<tr>
<th>Type</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child – 1 day</td>
<td>8</td>
</tr>
<tr>
<td>Adult – 1 day</td>
<td>6</td>
</tr>
<tr>
<td>Child – Season Pass</td>
<td>3</td>
</tr>
<tr>
<td>Adult – Season Pass</td>
<td>2</td>
</tr>
</tbody>
</table>

**H**

<table>
<thead>
<tr>
<th>Type</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child – 1 day</td>
<td>38</td>
</tr>
<tr>
<td>Adult – 1 day</td>
<td>36</td>
</tr>
<tr>
<td>Child – Season Pass</td>
<td>33</td>
</tr>
<tr>
<td>Adult – Season Pass</td>
<td>32</td>
</tr>
</tbody>
</table>

**G**

<table>
<thead>
<tr>
<th>Type</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child – 1 day</td>
<td>240</td>
</tr>
<tr>
<td>Adult – 1 day</td>
<td>180</td>
</tr>
<tr>
<td>Child – Season Pass</td>
<td>60</td>
</tr>
<tr>
<td>Adult – Season Pass</td>
<td>90</td>
</tr>
</tbody>
</table>

**J**

<table>
<thead>
<tr>
<th>Type</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child – 1 day</td>
<td>240</td>
</tr>
<tr>
<td>Adult – 1 day</td>
<td>180</td>
</tr>
<tr>
<td>Child – Season Pass</td>
<td>90</td>
</tr>
<tr>
<td>Adult – Season Pass</td>
<td>60</td>
</tr>
</tbody>
</table>
Carla combined a rectangle and a triangle to make the figure below.

What other information does Carla need to determine the area of this figure?

A  the length of w
B  the length of x
C  the length of y
D  the length of z
(Perason, 2013)

Appendix B

A “Knowledge Map” provided by the program

Appendix C

The “math” screen
Appendix D

The student progress chart

Appendix E

A student practice screen
References


Clara M., & B. (2013). Learning online, massive open online courses (MOOCs), connectivism, and cultural psychology. *Distance Education*, 129-136.


